Progress report on quantum gravity

Lee Smolin

1 Questions
2 Approaches
3 Basic assumptions of loop quantum gravity
4 Main results of loop quantum gravity to date
5 Applications
6 Present research directions

Sources: Rovelli, CUP +http://www.cpt.univ-mrs.fr/~rovelli/rovelli.html
Thiemann: gr-qc/0110034 + CUP (rigorous)
An invitation to loop gravity hep-th/0408048
Spin foam reviews: gr-qc/0301113,gr qc/0106091

Background independent approaches to quantum gravity:

- Annual meetings ~150 participants
- Major Groups:

Penn State
PI
Marseille
Utrecht
Orsay
Lyon
Berlin
Rome
SISSA
Imperial College
Nottingham
Beijing
Mexico City
Morelia
Montevideo
Maryland
LSU
Neils Bohr
Fredrickton

What are the questions?

1) **What is the structure of spacetime at Planck scales**

- Is spacetime discrete or continuous?
- If discrete, what are the elementary units of space or spacetime?
- What are the elementary events?
- What is the symmetry of the ground state?
- What is the spectrum of excitations?
- Is there any scaling at subplankian scales (asymptotic safety)

What are the questions?

1) What is the structure of spacetime at Planck scales

2) **What is fundamental and what is emergent?**

- Is space emergent or fundamental?

- Is locality emergent? Is there a fundamental notion of locality?

- Is time emergent or fundamental?

- Is causality emergent or fundamental?

- Is the lorentz group or other local gauge symmetries emergent or fundamental?

- Is matter emergent?

What are the questions?

1) What is the structure of spacetime at Planck scales

2) What is fundamental and what is emergent?

3) **What is the dynamics of the fundamental entities?**

4) **Does classical spacetime emerge?**

5) **What are the generic expectations for new experimentally accessible phenomena?**

- Modified dispersion relations/broken or modified Lorentz symmetry

- Early universe cosmology

- black hole entropy and spectra

Currently active approaches to these questions:

Background dependent approaches

(quantize fields or extended objects on fixed classical background space-times)

- String theory
- Asymptotic safety
- N=8 supergravity
- non-commutative geometry and field theory

Background independent approaches (no classical background spacetime):

- Derived from quantization of diffeomorphism invariant theories:

  LQG and spin foam models

- Models of background independent quantum physics

  causal sets
  Causal dynamical triangulations
  quantum graphity
  matrix models

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Loop quantum gravity: Four basic principles

Loop quantum gravity: Four basic principles

Loop quantum gravity is not a single theory. It is a method for quantizing diffeomorhism invariant theories of connections. These include general relativity and supergravity in all dimensions.

There are results from both Hamiltonian and path integral perspectives.

The basic physical principles

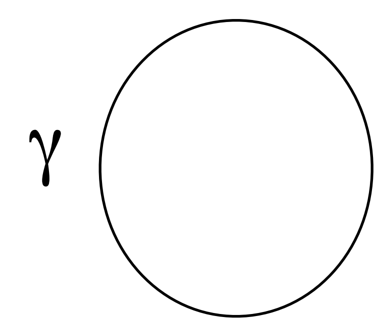
1) The Gauge principle: All forces are described by gauge fields

- Gauge fields: A_a valued in an algebra G
- Gravity: A_a valued in the lorentz group of SU(2) subgroup
- p form gauge fields
- Supergravity: Ψ_μ is a component of a connection.

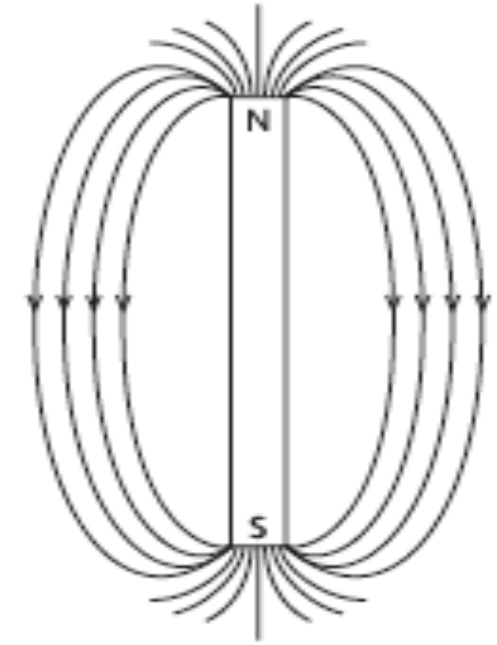
2) Duality: equivalence of gauge and loopy (stringy) descriptions

observables of gauge degrees of freedom are non-local:
described by measuring parallel transport around loops

Wilson loop $T[\gamma, A] = \mathrm{Tr}\, P \exp \int_\gamma A$



2) Duality: Quantum gauge fields can be described by the dynamics of their field lines.



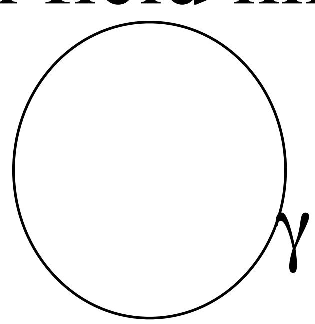
In a superconductor the magnetic fields are quantized and discrete.

In strong interaction physics the color electric field lines are quantized and discrete.

"Dual superconductor hypothesis"

In quantum gauge theories there are operators that create or annihilate individual field lines called Wilson loops

$T[\gamma,A] = \mathrm{Tr}\exp\int_\gamma A$



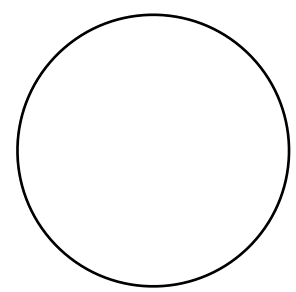
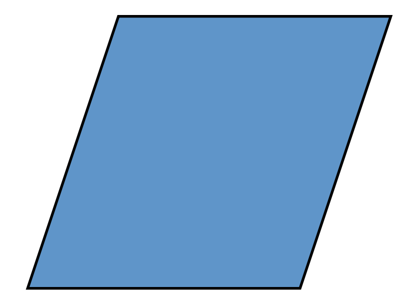
Other operators count field flux through surfaces.
The laws of motion of the field can be written completely in terms of these two operators.

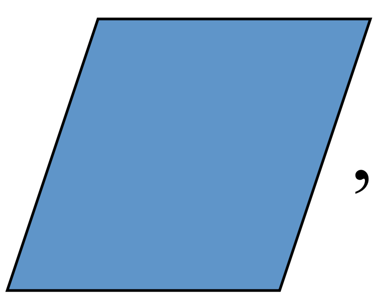
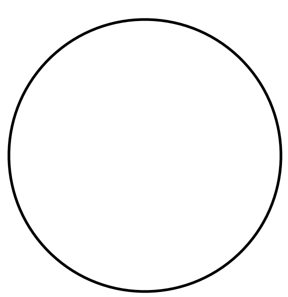
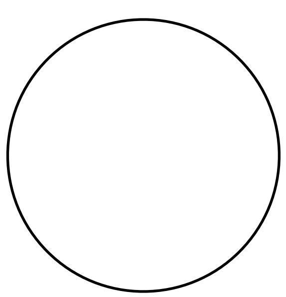
The gravitational field can be described as a gauge theory:

Spacetime connection = Gauge field = configuration variable
Spacetime metric = Electric field = momentum

- ***Quantum gauge fields can be described in terms of operators that correspond to Wilson loops and electric flux.*** These have a natural algebra that can be quantized:

The loop/flux algebra.

$T[\gamma,A] = \mathrm{Trexp} \int_\gamma A$  $E[S] = \int_S E$ 

[ , ] = h G Int 

Four principles:

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Developed on a background with fixed metric, these lead to string theory!

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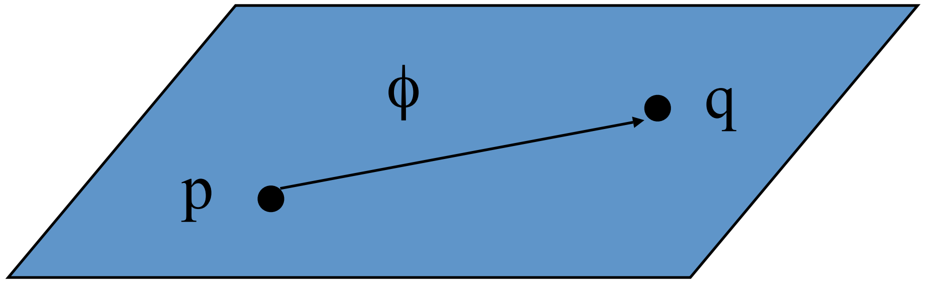
3) ***Diffeomorphism invariance and background independence***

3) Background independence (partial)

Means that are no fixed, non-dynamical fields and no global symmetries. Topology, differential structure and boundary conditions are fixed.

⇒ gauge invariance of general relativity includes ACTIVE diffeomorphism invariance of the spacetime manifold.



⇒ spacetime is NOT modeled by a manifold and metric, but by the equivalence class of manifolds and metrics, which are equivalent under any diffeomorphism !! Points are only distinguished by values of fields.

⇒ Realizes the basic principle that space and time are not fixed but reflect only dynamically evolving relationships

4) Locality:

Four principles:

1) The Gauge principle: All forces are described by gauge fields

2) Duality: equivalence of gauge and loopy (stringy) descriptions

3) Background independence and diffeomorphism invariance.

4) Locality

Basic results

LQG depends on a basic insight into the dynamics of diffeomorphism invariant field theories, common to GR, supergravity etc.

For quantization we want to put the field theory into the simplest form possible. For ordinary QFT's this is a low order polynomial which can be understood as a collection of weakly coupled harmonic oscillators. This can't work for a diffeo invariant theory as there is no anomoly free rep of the diffeo group on Fock space. Furthermore if we try to represent GR as an expansion around a free field theory there are infinite order interactions.

Is there an alternative?

The simplest form of a non-linear theory is given by quadratic equations, hence by a cubic action. This means no more complicated operator products than quadratic in quantum field equations.

We can write a cubic action for GR, supergravity etc, by adding Lagrange multipliers. This gives us one meaning of locality.

This leads to an understanding that GR is closely related not to free field theory but to topological field theory.

The Plebanski action

Consider a theory of an SU(2) connection A^{AB} on a four manifold ***M***, with a 2-form field $B^{AB,}$ both valued in SU(2)

$$S^{Pleb} = \frac{1}{G}\int B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i$$

This is a topological field theory, with no local dof.
The equations of motion are:

$$F_i = \Lambda B_i$$

$$\mathcal{D} \wedge B^i = 0$$

The Plebanski action

Add a term, with Φ_{ij} a spin two field (symm and trace-free)

$$S^{Pleb} = \frac{1}{G}\int B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i \quad + \phi_{ij} B^i \wedge B^j$$

This is now an action principle for general relativity.
The equations of motion are:

$$F_i = \Lambda B_i \; - \phi_{ij} B^j$$

$$\mathcal{D} \wedge B^i = 0$$

$$B^i \wedge B^j - \frac{1}{3}\delta^{ij} B^k \wedge B_k = 0$$

There exists frame fields e^a $s.t.$ $B^i =$ self-dual part of $[e^a \wedge e^b]^i$

So the configuration space is a space of SU(2) connections on a 3-manifold

The metric components are canonical momenta

$$S^{Pleb} = \frac{1}{G}\int B^i \wedge F_i - \frac{\Lambda}{2} B^i \wedge B_i \quad + \phi_{ij} B^i \wedge B^j$$

$$= \frac{1}{G}\int dt \int_{\pm} E^i \wedge \partial_0 A_i - A_{0i} \mathcal{D} \wedge E^i \ldots$$

$$E^i_{bc} = \text{self-dual part}[e \wedge e]^i_{bc}$$

$$\Rightarrow \qquad \{A^i_a(x), E_{bcj}(y)\} = G\epsilon_{abc}\delta^i_j \delta^3(x,y)$$

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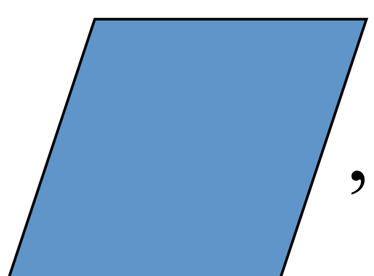
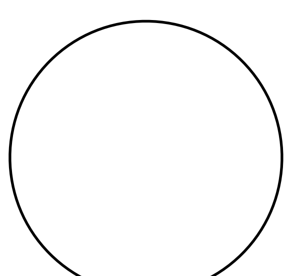
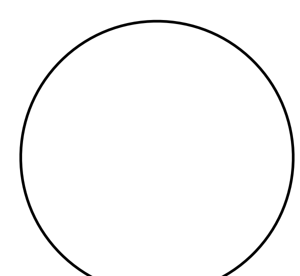
[ , ] = h G Int 

The fundamental theorem of loop quantum gravity:

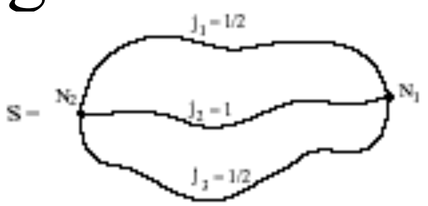
The LOST theorem:

For any compact G, there is a unique representation of the algebra of Wilson loops and electric fluxes in d>1 spatial dimensions which carries a non-anomalous representation of the spatial diffeomorphism group, and in which the electric field flux and magnetic field flux through finite surfaces are represented by finite, well defined operators.

Lewandowski, Okolo, Sahlmann, Thiemann+ Fleishhack   2003

More precisely: the unique Hilbert space is defined as follows:

Def: G-spin network is a graph Γ with edges labeled by spins (representations of G) a vertices labeled by invariants.



Given any Γ there is an extension of a Wilson loop, gotten by tracing the parallel transports of the connection on the edges in the representations given by the labels.

Theorem:

- **The unique Hilbert space, H^{kin} has an orthonormal basis in 1-1 correspondence with the embeddings of spin nets Γ in Σ.**

Physical states must be diffeomorphism invariant.
These must satisfy:

` $$\Psi[\Gamma] = \Psi[diffeo \quad \Gamma]$$

Using the unitary rep of diffeo's we can construct these exactly.

- The diffeomorphisms of Σ give unitary operators on H^{kin} :

$$U\ (\phi)\ |\ \Gamma > \ = \ |\ \phi^{-1}\ \ \Gamma> \qquad \phi \text{ in } Diff(\Sigma\ ) \text{ piecewise smooth}$$

Example: $\Psi[\Gamma] = K[\Gamma]$

where $K[\Gamma]$ be an invariant of knotted graphs, i.e the Jones polynomial

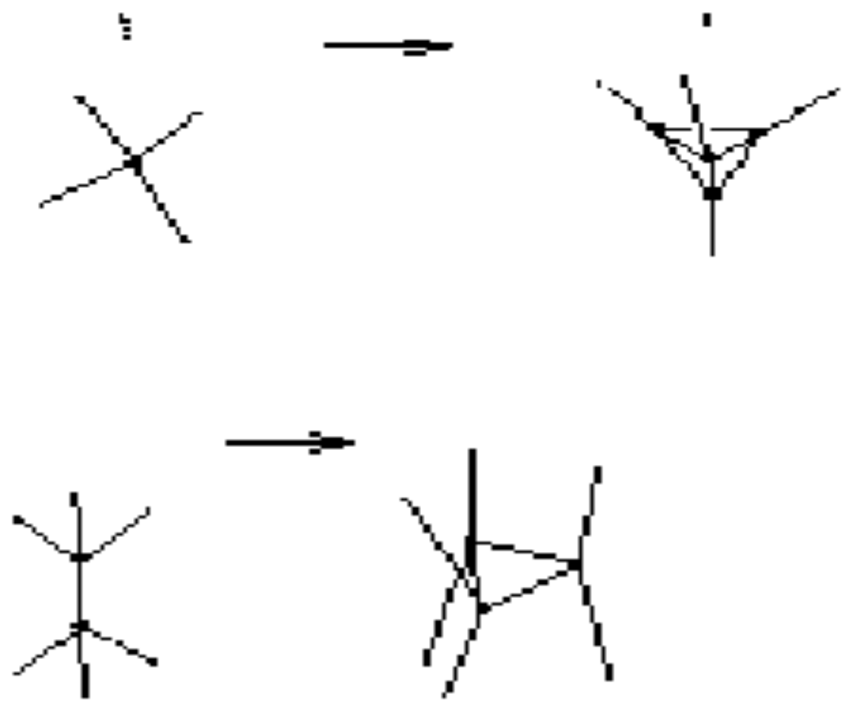
- The space of diffeomorphism invariant states, H^{diff} , has an orthonormal basis labeled by $\{\Gamma\}$, the diffeo classes of embeddings of all spin networks Γ in Σ.

$$< \{\Gamma\}|\Gamma' > = 1 \ \text{ if } \Gamma' \text{ is in } \{\Gamma\} \qquad 0 \text{ otherwise.}$$

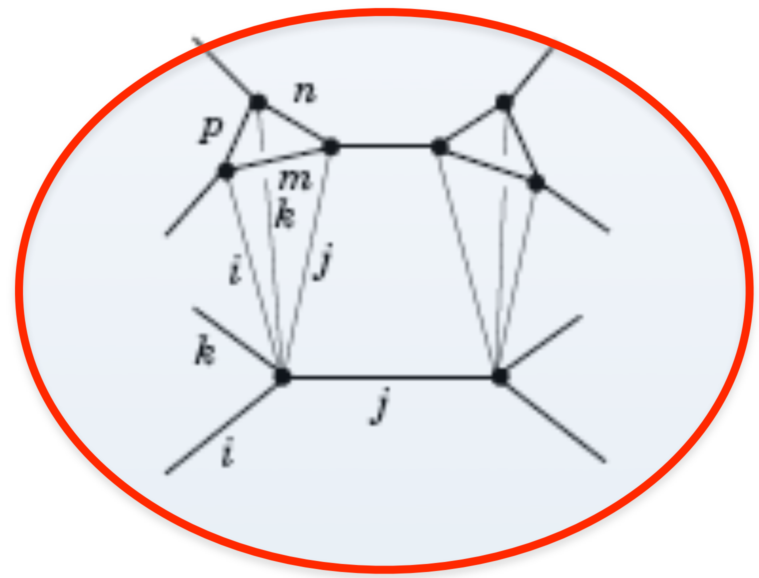
- H^{diff} is separable with piecewise smooth diffeos

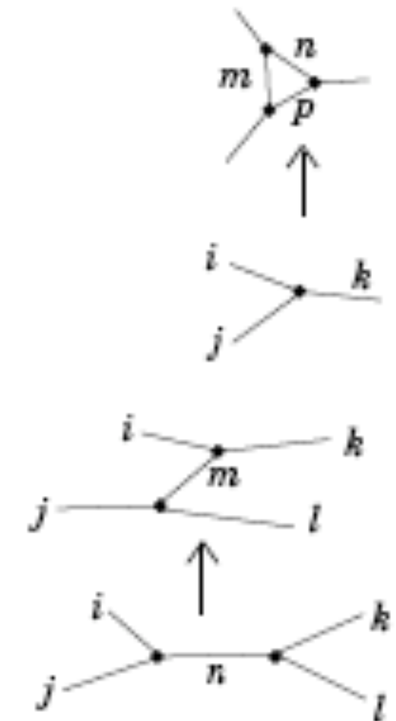
- Inner product: $< \{\Gamma\}|\{\Gamma'\} > = \delta_{\{\Gamma\}\{\Gamma'\}}$

4) Locality: the dynamics is generated by local moves in the graph



Generating moves for
four valent graphs





Generating moves for
three valent graphs

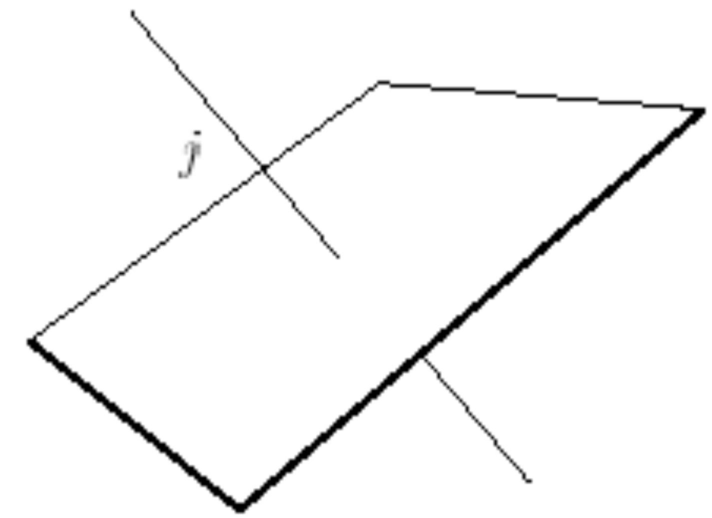
A theory gives amplitudes to these moves which are functions of the labels. The amplitudes corresponding to the quantization of general relativity are known in detail. They can be constructed by constraining the path integral for BF theory.

GEOMETRIC OBSERVABLES ARE FINITE AND DISCRETE

Let the spatial manifold, Σ have a boundary, B.

Some observables can be constructed via a regularization procedure that respects diffeomorphism invariance. These are ***finite*** and have ***discrete spectra:***

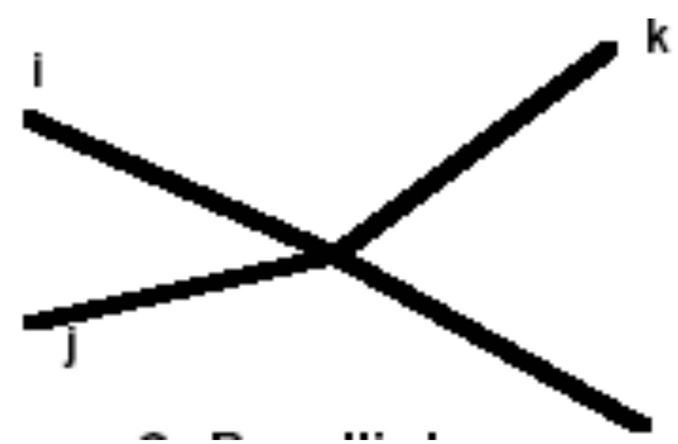
- Volume of Σ
- Area of the boundary of Σ
- Hamiltonian constraint



$8\pi\gamma l_P^2 \sqrt{j(j+1)}.$

$$\textbf{Area} = hG\ \Sigma_j \sqrt{j(j+1)}$$

$$\textbf{Volume} = l^3_{Pl}\ \Sigma_j\ v_j$$



•Hence there is a smallest physical volume and area.
•Each spinnet state can be interpreted as a discrete quantum geometry with quantized volumes on nodes and areas on edges.

In this theory states are given by networks of field lines

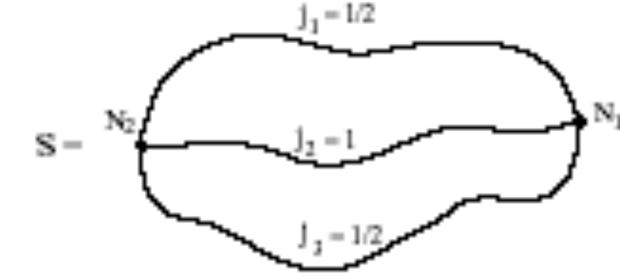
Geometry is measured in terms of the properties of the field lines.

Area through a surface = electric flux through the surface

Volume in a region ~ number of nodes in the field lines.

HENCE: Quantum geometry is discrete because field lines are discrete.

The basic physical picture:



•The gravitational field is a gauge field.

•We quantize it in a representation that realizes quantization of electric flux. (This has been proved to be the only possibility consistent with the gauge and diffeomorphism symmetries.)

The states are graphs: networks of quantized electric flux.

•This implies discreteness of all areas and volumes, so the t is ultraviolet finite.

•The gauge invariance includes diffeomorphism invariance. there is no meaning to where the graphs are in spac their topology matters.

•Dynamics is local moves in the connectivity of the graphs.

But what about perturbative non-renormalizability?

That depends on two assumptions:

- Spacetime is smooth at arbitrarily short distances
- Lorentz invariance holds as usual at arbitrarily high energies.

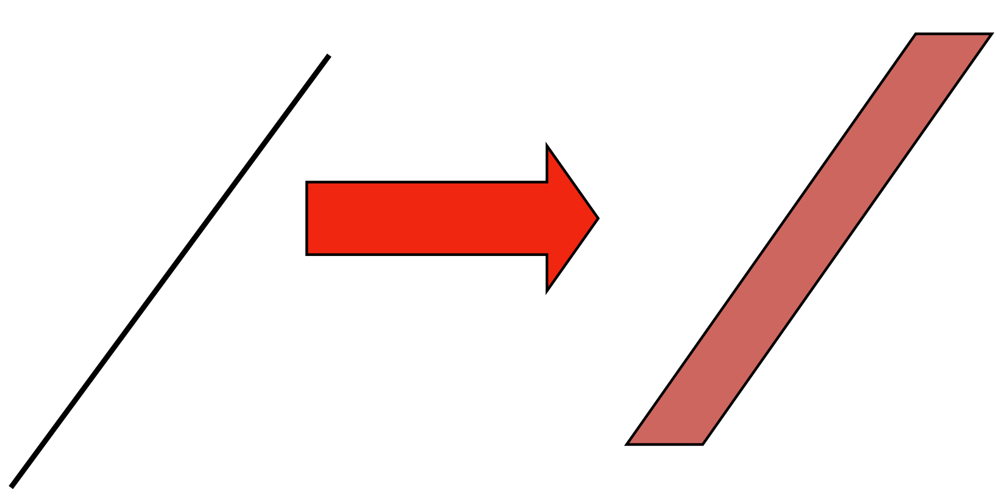
Neither of these are implied by the combination of general relativity and quantum field theory. In fact, that combination implies the negation of the first, and perhaps the second!

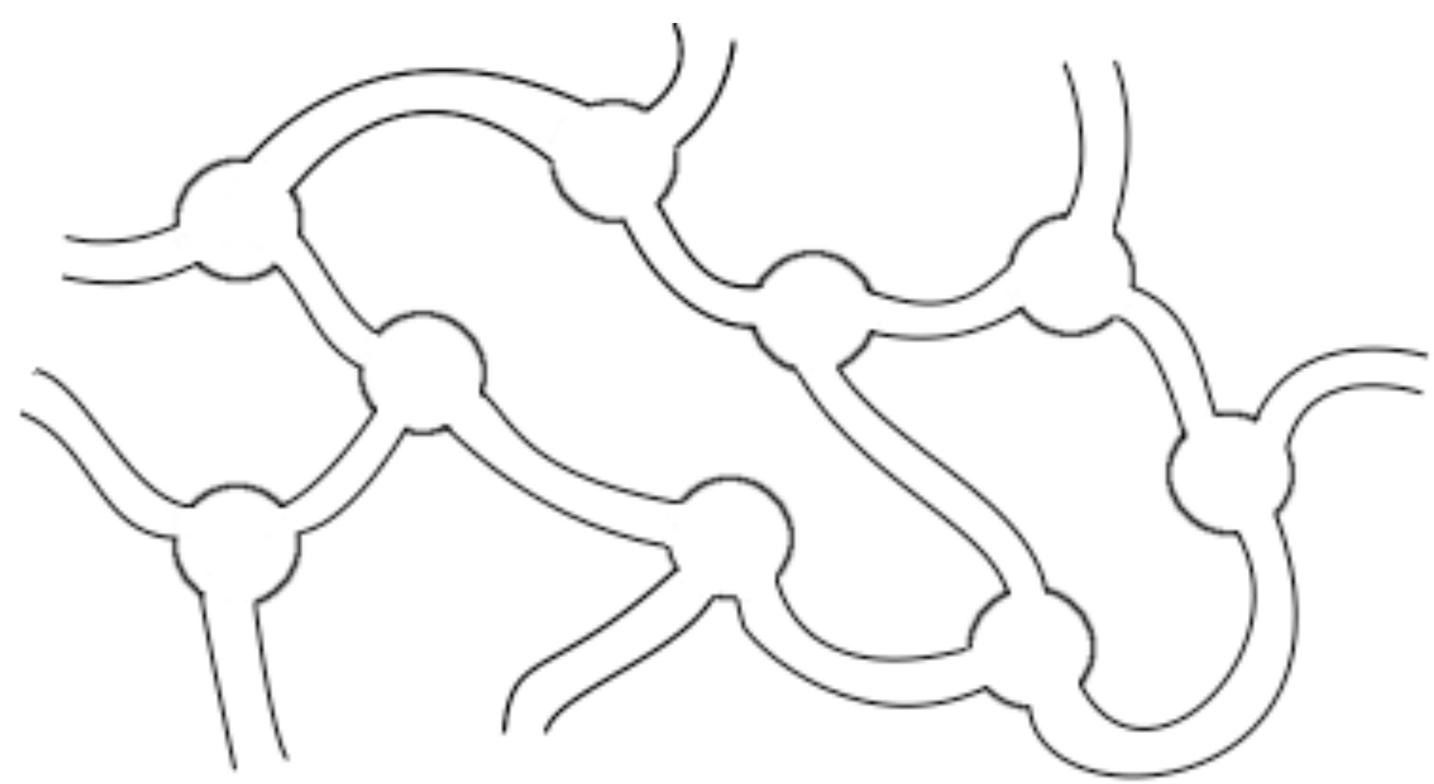
Role of the cosmological constant:

Requires quantum deformation of SU(2)

$$q = e^{2\pi i/k+2} \qquad k = 6\pi/G\Lambda$$

To represent this the spin network graphs must be framed:





Basic dynamical results (hamiltonian theory)

- existence and finiteness of hamiltonian constraint operator after regularization consistent with spatial diffeo invariance.

- consistency of quantum constraint algebra

- infinite numbers of exact solutions to all quantum constraints

- semiclassical states for flat space and (A)deSitter spacetime.

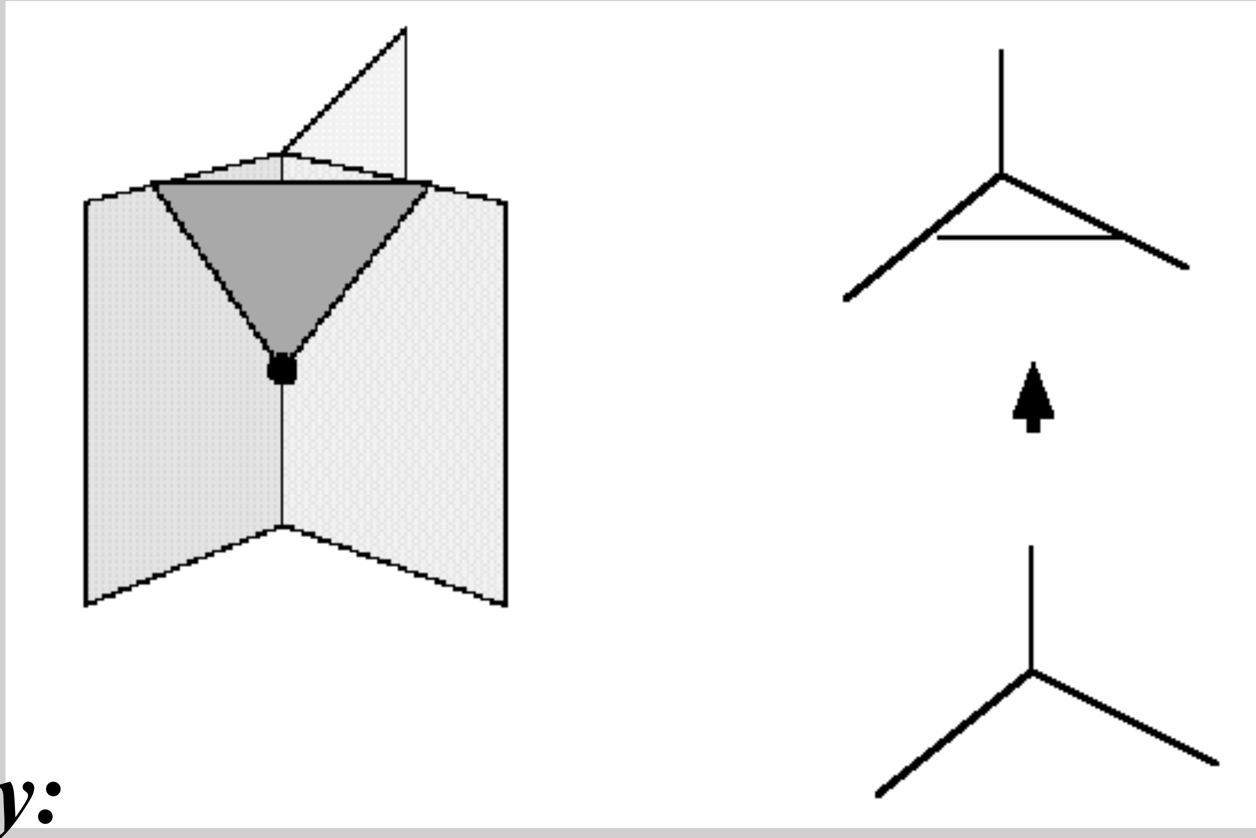
- excitations are massless spin 2 gravitons

- there are conserved chiral excitations which propagate and interact (possible matter)

Basic dynamical results: path integrals or spin foams

A spin foam can be expressed as:

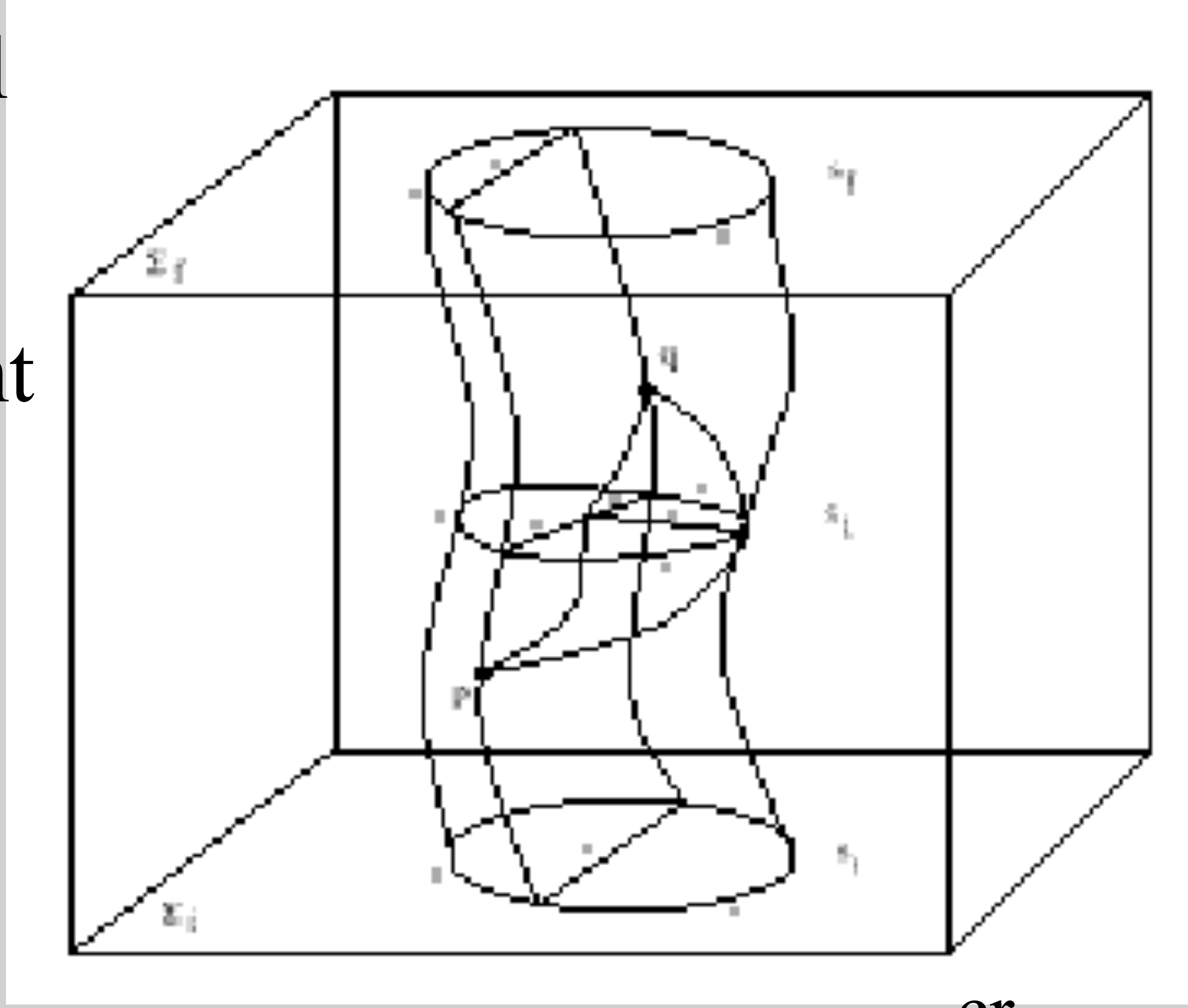
•A sequence of local moves
•A causal (partially ordered) set
(as a discrete causal structure)
•As a complex of labeled 2-surfaces



Spin foam amplitudes can be derived by:

•Deriving the amplitudes for a topological field theory and then imposing the constraints in the measure
•Exponentiating the hamiltonian constraint

•By expressing them as the Feynman diagrams of a certain matrix model
All approaches give the same general theory



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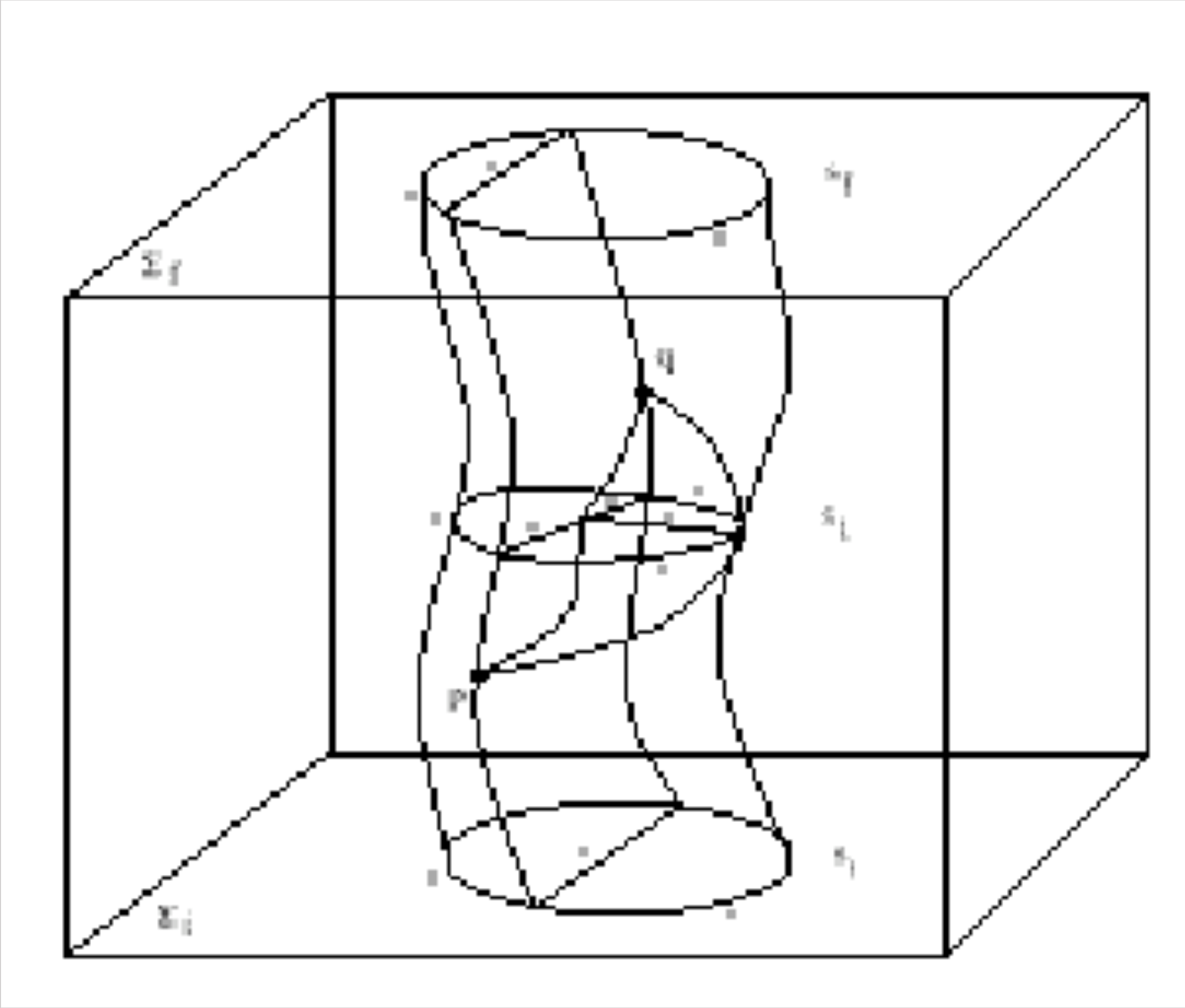
The sum over spin foams is like
a Feynman diagram computation.

•Rather than initial and final momentum eigenstates we have initial and final spin networks.

•Rather than diagrams we have spin foam histories.

•Rather than summing over momenta on edges
we sum over spins on faces.

Everything is one dimension up



For each spin foam model there is a matrix model such that the spin foams are the Feynman diagrams of that model.

Basic dynamical results: path integrals or spin foams

- Existence of path integrals
  Closed form path integral for quantum BF theory
  Quantum GR amplitudes in closed form from constraining BF measure.
  Also obtained by discretization of formal path integral.
  uv finiteness of some forms.
  Expression in terms of matrix models (GFT)

- Semiclassical approximation
  Dominated by Regge calculus in some limits.

- Propagator for spin 2 computed and agrees with semiclassical theory.

Applications:

- Black holes and cosmological horizons
- Coupling to matter
- 2+1 gravity coupled to matter
- Cosmology
- Phenomenology

Results on black holes and horizons

Horizons can be treated as boundaries.

- There is a boundary action and boundary conditions that are satisfied at horizons (isolated horizons.)

-The boundary theory is a Chern-Simons theory

$$S^{Pleb} = \int_{\mathcal{M}} B^i \wedge F_i - \ldots - \frac{k}{4\pi} \int_{\partial\mathcal{M}} Y_{CS}(A)$$

Boundary condition:

$$B^i|_{\partial\mathcal{M}} = E^i|_{\partial\mathcal{M}} = \frac{k}{2\pi} F^i|_{\partial\mathcal{M}}$$

Denotes vanishing expansion of null geodesics:
k ~ surface gravity for black hole, for cosmological horizon $k = \dfrac{6\pi}{\hbar G \Lambda}$

The horizon is a punctured sphere, with punctures at the ends of spin network edges, so we get a relation between area and the dimension of the boundary state space

Results on black holes and horizons:

•There are exact results for boundary conditions of the form

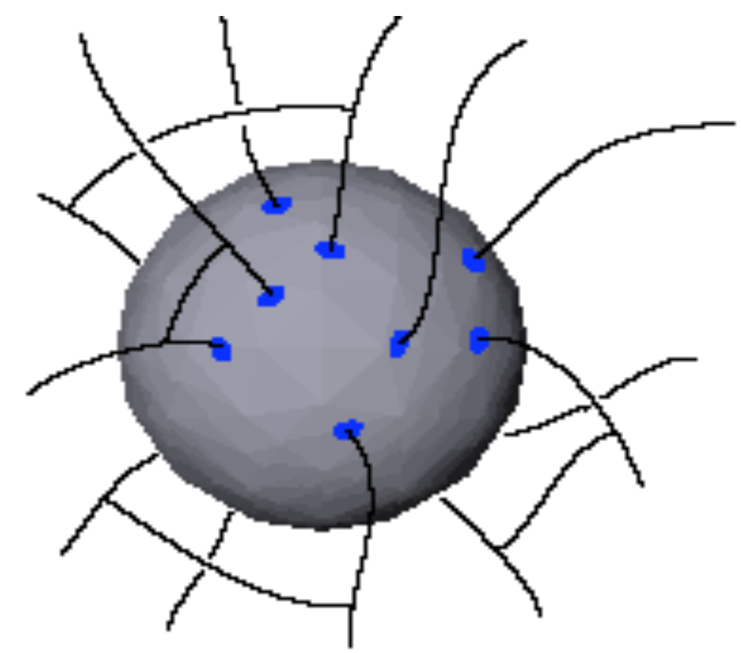
E (s.d. 2-form of metric) = constant F (left-handed curvature)

•This includes all black hole and cosmological horizons

•The horizon state space is described in terms of Chern-Simons theory

•It can be decomposed into eigenstates of area

•For each area the horizon state space is finite dimensional-gives the exact quantum geometry of the horizon.



•The Bekenstein Hawking relation holds exactly

$$\mathbf{S\ (entropy) = Area\ /\ 4Gh}$$

•The (finite) renormalization of G required is fixed independently by matching to the quasi normal mode spectrum of bh's (Dreyer)

Black hole singularities removed:

Modesto gr-qc/0407097, 0504043
Husain & Winkler gr-qc/0410125
Ashtekar & Bojowald gr-qc/0504029

In models of the interior geometry, by the same mechanism as works in quantum cosmological models.

Corrections to Hawking radiation:

Ansari hep-th/0607081

Corichi, Diaz-Polo,Fernandez-Borja
gr-qc/0609122

Coupling to matter

- Most major results extend to coupling to gauge, spinor and scalar fields

- Most major results extend to N=1 SUGRA, some to N=2

- Unification arises by extending gauge group in Plebanski formalism

$$SU(2) \rightarrow G \subset SU(2) + H$$

- Some results on chiral excitations and standard model.

Dimensional reduction to 2+1 gravity

- quantum theory of 2+1 pure gravity a tqft, exactly solved.

- 2+1 coupled to matter: symmetry of the ground state is quantum deformed to kappa-poincare algebra.

Applications to cosmology

The big question: is the singularity at the big bang still there or was there time and a universe before the big bang?

To study the very early universe a class of models is constructed by reducing to homogeneous spacetimes and then quantizing the reduced theory using the methods of LQG. These are loop quantum cosmology models (LQC).

These models can be solved exactly and the result is that the singularities are removed.

Basic LQC results:

- The ***exact*** evolution by the Hamiltonian constraint solved, numerically, in terms of effective equations and in some cases exactly.

- Coupling to matter, including inflaton fields has been incorporated exactly.

- At large volumes (in planck units) FRW classical cosmology is recovered

- At small volumes the singularity is absent, and replaced by a bounce, before which the universe was contracting.

- At small volume leading corrections have been studied and understood.

Cosmological singularities are replaced by bounces:

Big Crunch Avoidance in k = 1 Semi-Classical Loop Quantum Cosmology

Parampreet Singh*, Alexey Toporensky†

gr-qc/0312110

Gravity coupled to a massive scalar field:

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 \ .$$

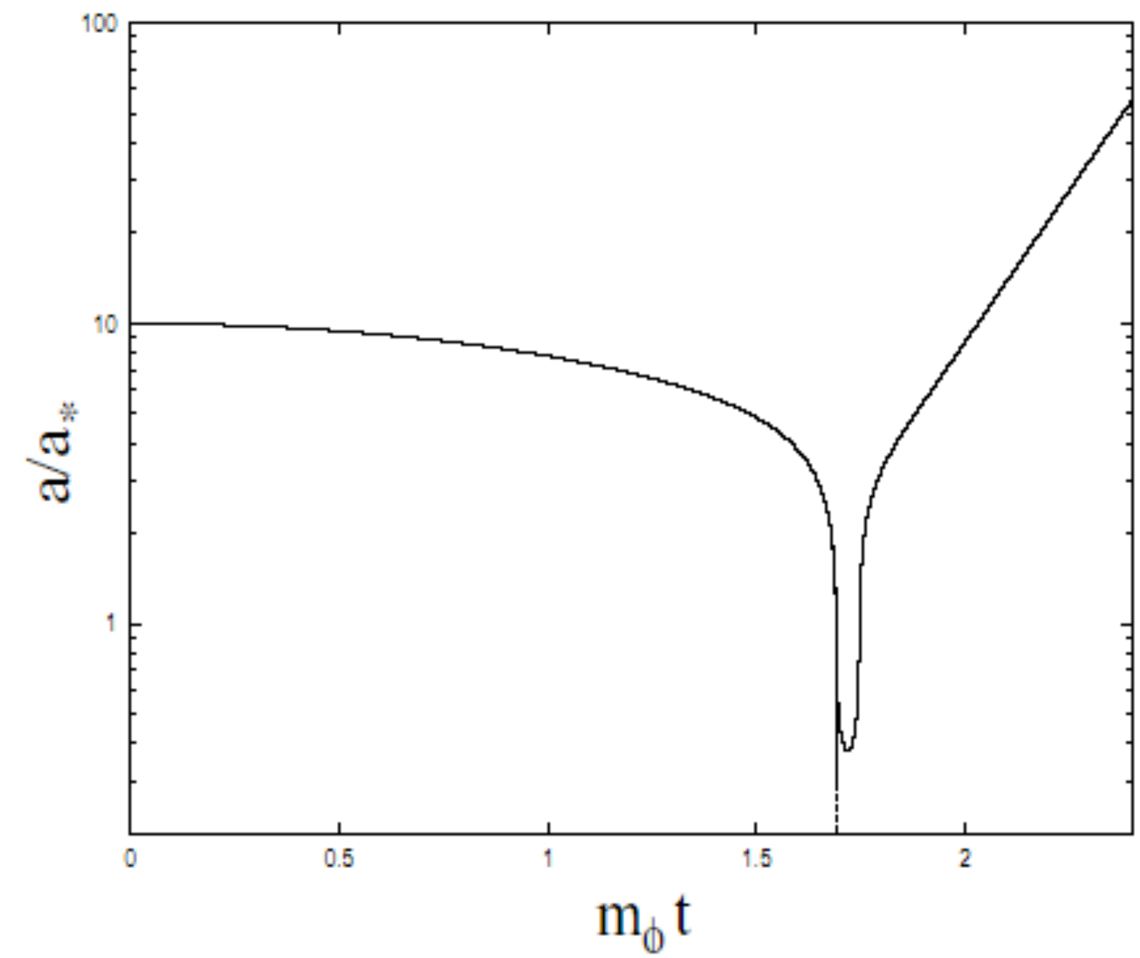
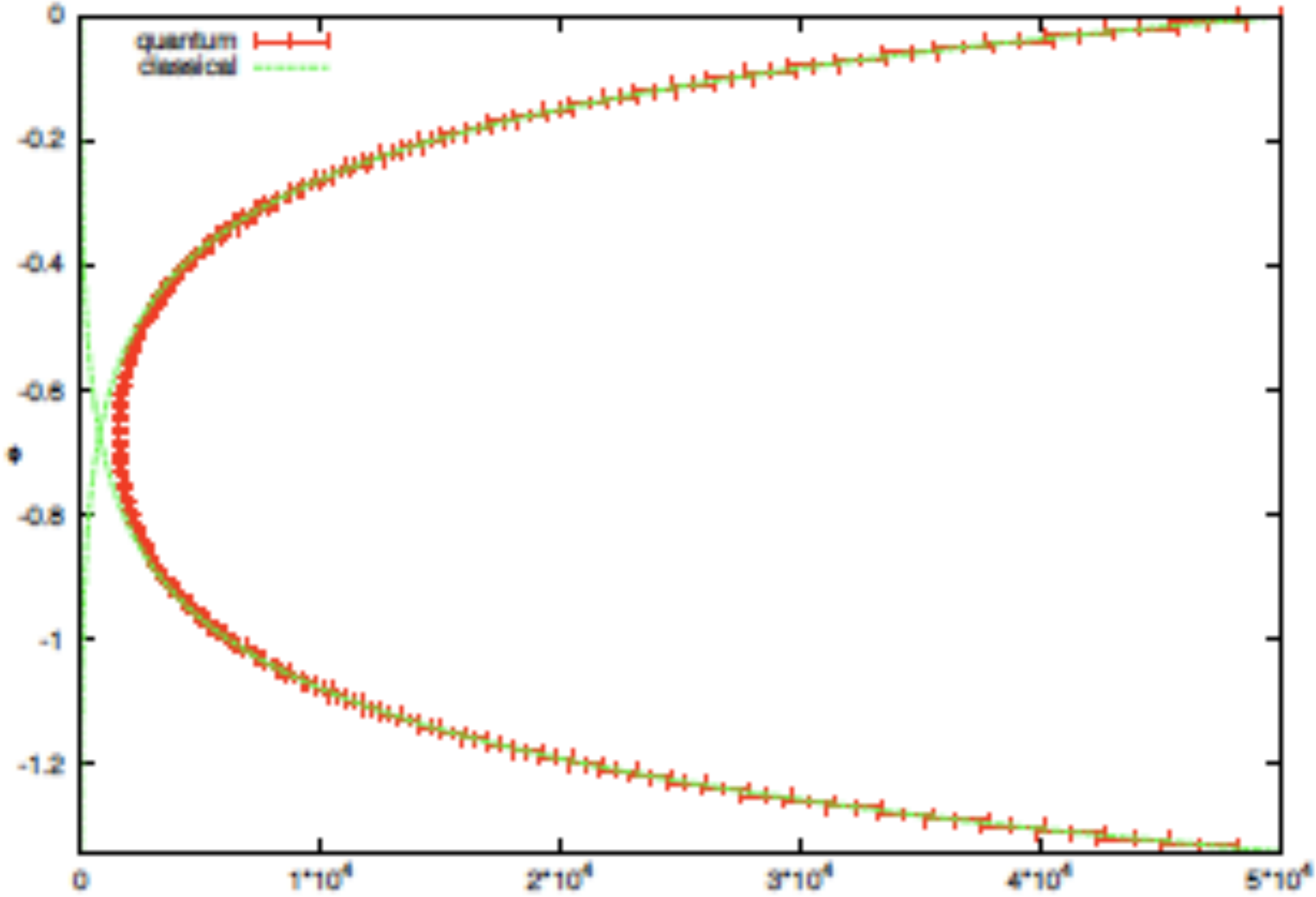


FIG. 1: Evolution of scale factor for a closed universe in classical cosmology is shown by the dashed curve and in loop quantum cosmology is shown by the solid curve for the quadratic potential with $j = 100$, $m_\phi = 0.1 M_{\rm pl}$ and initial conditions $a_i = 10\, a_*, H_i = 0, \phi_i = 0$ and $\dot{\phi}_i$ determined from eq.(5). A collapsing closed universe in classical cosmology encounters a big crunch whereas in loop quantum cosmology it bounces into an expanding phase when $a = 0.38\, a_*$ and avoids big crunch.

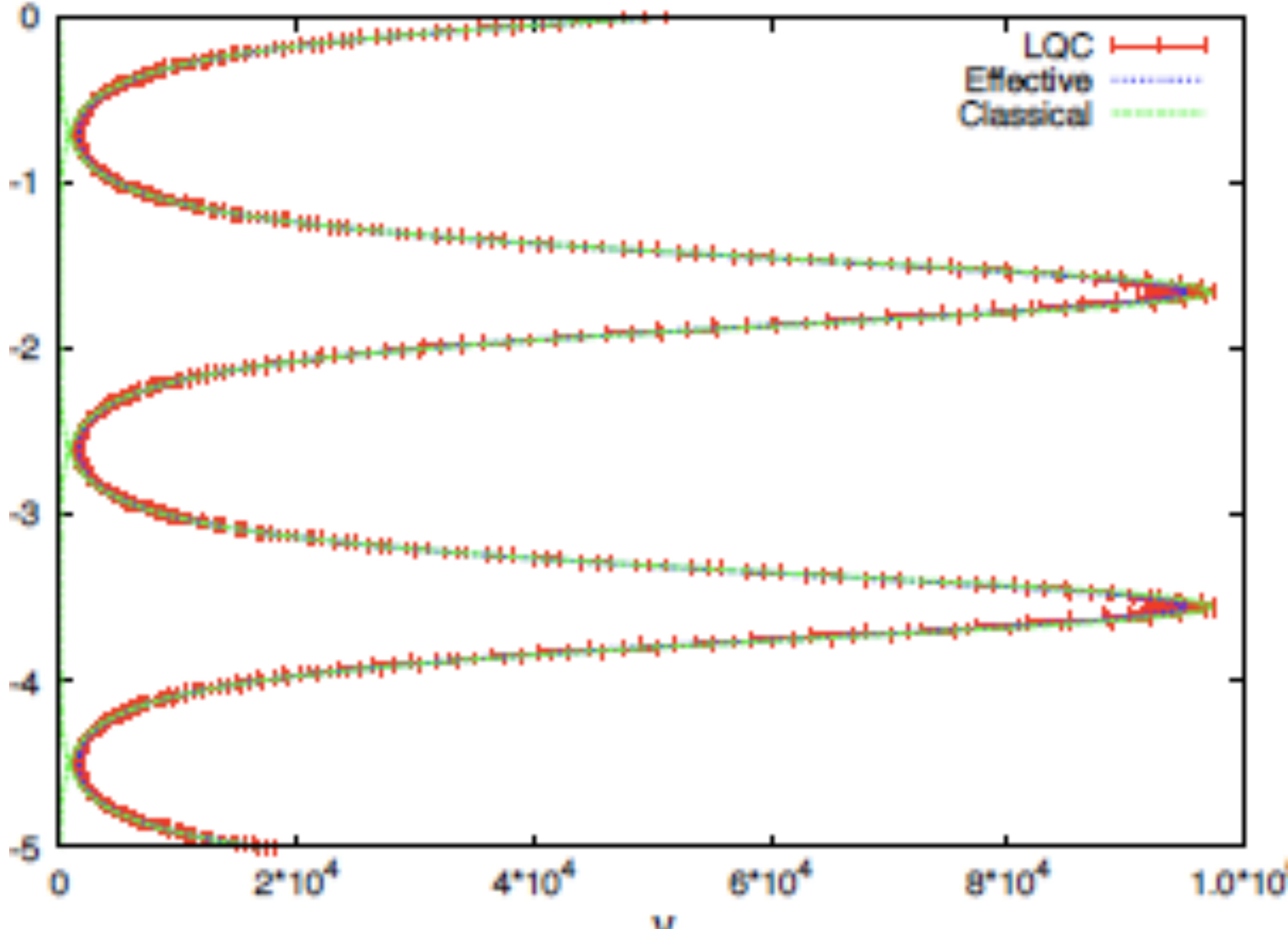
Ashtekar et al

k=0 quantum cosmology



Ashtekar et al

k=1 quantum cosmology



Are there consequences for cosmological observations?

•Inflation is recovered by coupling to scalar field.

•Corrections to the power spectrum for inflation derived

Hofman +Winkler astro-ph/0411124, Hossain gr-qc/04110124

There is an order L_p term!!

Gives a 10% effect for quadrapole mode

Loop quantum gravity effects on inflation and the CMB

Shinji Tsujikawa[1], Parampreet Singh[2], and Roy Maartens[1]

astro-ph/
0311015

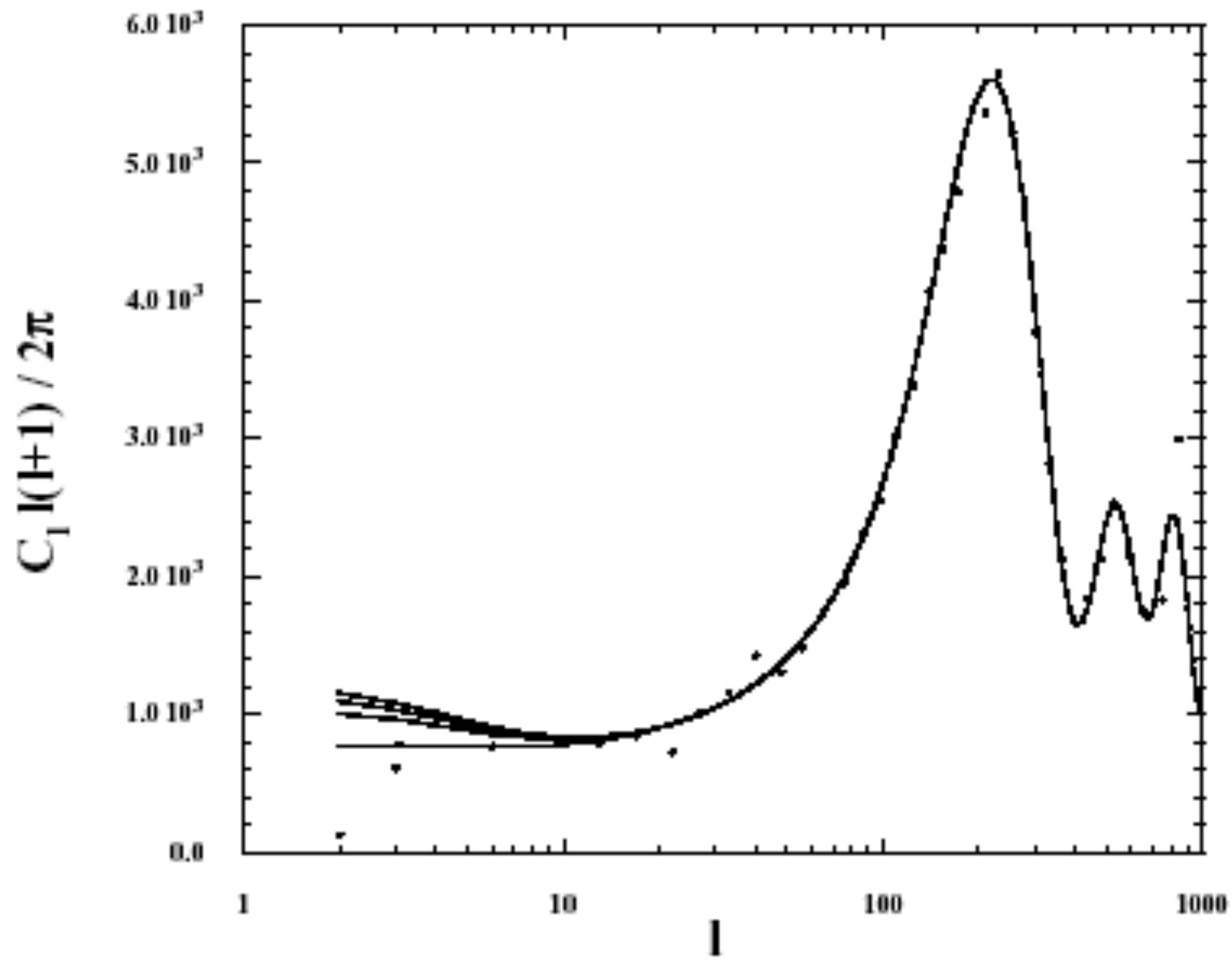


FIG. 3: The CMB angular power spectrum with loop quantum inflation effects. From top to bottom, the curves correspond to (i) no loop quantum era (standard slow-roll chaotic inflation), (ii) $\bar{\alpha} = -0.04$ for $k \le k_0 = 2 \times 10^{-3}\,\mathrm{Mpc}^{-1}$, (iii) $\bar{\alpha} = -0.1$ for $k \le k_0$, and (iv) $\bar{\alpha} = -0.3$ for $k \le k_0$. There is some suppression of power on large scales due to the running of the spectral index.

Phenomenology

Phenomenology

- The main question is what is the symmetry of the ground state?
    - Poincare invariance?
    - Broken Poincare invariance?
    - Deformed poincare invariance? (DSR, the subject of tomorrow's talk)

- Does LQG make predictions for these?
    - No definitive or rigorous results.

    - In 2+1 gravity with matter, the answer is DSR in the form of a quantum deformation of Poincare symmetry called kappa-Poincare symmetry.

    - In 3+1 there are heuristic, semiclassical arguments for DSR, in the form of an energy dependence of the metric arising in the semiclassical approximation.

Arguments for DSR from the semi-classical limit of quantum gravity

- **Not rigorous**
- **Two**

ls hep-th/0501091, Nucl.Phys. B742 (2006) 142-157.
ls arXiv:0808.3765

Why should the metric become energy dependent?

Variables: $A_a^i, \quad \tilde{E}_i^a, \quad qq^{ab} = \tilde{E}_i^a \tilde{E}_j^b \delta^{ij}$

Poisson brackets: $\{A_a^i(x), \tilde{E}_j^b(y)\} = G\delta_a^b \delta_j^i \delta^3(x,y)$

Connection rep: $\Psi(A,\phi) \qquad \hat{\tilde{E}}_i^a(x) = G\hbar \dfrac{\delta}{\delta A_a^i(x)}$

ϕ matter fields

Variables: $A_a^i, \quad \tilde{E}_i^a, \quad qq^{ab} = \tilde{E}_i^a \tilde{E}_j^b \delta^{ij}$

Poisson brackets: $\{A_a^i(x), \tilde{E}_j^b(y)\} = G\delta_a^b \delta_j^i \delta^3(x,y)$

Connection rep: $\Psi(A,\phi) \qquad \hat{\tilde{E}}_i^a(x) = -\imath G\hbar \dfrac{\delta}{\delta A_a^i(x)}$

Semiclassical states: $\Psi(A,\phi) = e^{iS(A)}\xi(A,\phi)$

S: Hamilton-Jacobi function

classical solution: $\tilde{E}_0^{ai}(x) = \dfrac{\delta S}{\delta A_{ai}(x)}$

S(A) is a time coordinate on configuration space and on solutions S=μ T where T is a coordinate on the spacetime

So an energy eigenstate $\xi[T,\phi] = e^{-\imath\omega T}\xi_\omega[\phi]$

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution

a_{ai}- fluctuations of metric, we ignore them.

$$\begin{aligned}\hat{\tilde{E}}{}_i^a(x)\xi[\mathcal{A},\phi] &= -\imath\hbar\rho\frac{\delta\xi[\mathcal{A},\phi]}{\delta\mathcal{A}_a^i(x)} \\ &= \left(-\tilde{E}_i^{0a}\frac{\imath\hbar\rho}{M}\frac{\delta}{\delta\mathcal{S}(x)} - \imath\hbar\rho\frac{\delta}{\delta a_{ai}(x)}\right)\xi[\mathcal{S},a_{ai},\phi]\end{aligned}$$

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution

$$\hat{\tilde{E}}_i^a(x)\xi[\mathcal{A},\phi] = \tilde{E}_i^{0a} \frac{-\imath\hbar\rho}{\mu M} \frac{\partial}{\partial T} \xi[T,\phi]$$

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution:

$$\hat{\tilde{E}}_i^a(x)\xi[\mathcal{A},\phi] = \tilde{E}_i^{0a}\frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi] = -\tilde{E}_i^{0a}\alpha l_{Pl}\omega e^{-\imath T\omega}\xi_\omega[\phi]$$

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution:

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

$$\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] = \tilde{E}^{0a}_i \frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi] = -\tilde{E}^{0a}_i \alpha l_{Pl}\omega e^{-\imath T\omega}\xi_\omega[\phi]$$

Putting everything together

$$\hat{\tilde{E}}^a_i(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}^{0a}_i\left(1 - \alpha l_{Pl}\omega\right)\xi[T,\phi]$$

Classical term

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution:

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

$$\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] = \tilde{E}^{0a}_i \frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi] = -\tilde{E}^{0a}_i \alpha l_{Pl}\omega e^{-\imath T\omega}\xi_\omega[\phi]$$

Putting everything together

$$\hat{\tilde{E}}^a_i(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}^{0a}_i\left(1 - \alpha l_{Pl}\omega\right)\xi[T,\phi]$$

Energy dependent correction

Semiclassical states:

$$\Psi(A,\phi) = e^{iS(A)} e^{-\imath\omega T} \xi_\omega[\phi]$$

Decompose E operator around a solution:

$$\frac{\hbar\rho}{M\mu} = \alpha l_{Pl}$$

$$\hat{\tilde{E}}^a_i(x)\xi[\mathcal{A},\phi] = \tilde{E}^{0a}_i \frac{-\imath\hbar\rho}{\mu M}\frac{\partial}{\partial T}\xi[T,\phi] = -\tilde{E}^{0a}_i \alpha l_{Pl}\omega e^{-\imath T\omega}\xi_\omega[\phi]$$

Putting everything together

$$\hat{\tilde{E}}^a_i(x)\Psi_0[\mathcal{A}]\xi[T,\phi] = \Psi_0[\mathcal{A}]\tilde{E}^{0a}_i\left(1-\alpha l_{Pl}\omega\right)\xi[T,\phi]$$

So the spacetime metric has become energy dependent

$$g \longrightarrow g(\omega) = -dT\otimes dT + \sum_i e_i \otimes e_i (1-\alpha l_{Pl}\omega)$$

And there is a modified dispersion relation to leading order:

$$m^2 = -g(\omega)^{\mu\nu}k_\mu k_n u = \omega^2 - \frac{k_i^2}{(1-\alpha l_{Pl}\omega)} + O[(l_{Pl}\omega)^2]$$

Second argument for DSR from the semiclassical limit of quantum gravity

•Semi-classical general relativity, a new way. Rather than posit:

$$G_{ab} - \Lambda g_{ab} = 8\pi G < \xi|\hat{T}_{ab}|\xi >$$

•Use Λ = 1/L^2 as an infra red regulator and get the flat spacetime behavior by taking the limit as $\Lambda \rightarrow 0$, scaling operators by appropriate powers of (L/l_P).

•Study the reaction of the conformal mode to quantum energy density

$$ds^2 = -dt^2 + e^{\sqrt{\frac{\Lambda}{3}}t+\phi}\delta_{ab}dx^a dx^b$$

•Promote the conformal mode to an operator on the QFT Hilbert space defined by solving the Hamiltonian constraint with no gravitons

$$(\nabla^2 - \frac{\Lambda}{3})\phi = 4\pi G\hat{T}_{00} \qquad \rightarrow \hat{\phi} = (\nabla^2 - \frac{\Lambda}{3})^{-1} 4\pi G\hat{T}_{00}$$

Hence, in a state |ϕ> the expectation value of the metric is

$$\langle\xi|ds^2|\xi\rangle = -dt^2 + e^{\sqrt{\frac{\Lambda}{3}}t+\langle\xi|\frac{4\pi G\hat{T}_{00}}{(\nabla^2 - \frac{\Lambda}{3})}|\xi\rangle}\delta_{ab}dx^a dx^b$$

$$\langle\xi|ds^2|\xi\rangle = -dt^2 + e^{\sqrt{\frac{\Lambda}{3}}t + \langle\xi|\frac{4\pi G\hat{T}_{00}}{(\nabla^2 - \frac{\Lambda}{3})}|\xi\rangle}\delta_{ab}dx^a dx^b$$

Now, put the matter in a one particle energy Eigenstate:

$$\hat{H}|k\rangle = \hbar\omega_k|k\rangle \quad \rightarrow \hat{T}_{00}|k\rangle = \frac{\hbar\omega_k}{L^3}|k\rangle$$

Since the state is a momentum eigenstate

$$\nabla^2 < k|\hat{T}_{00}|k >= 0$$

Hence, we have an effective metric operator whose expectation value is

$$\langle k|ds^2|k\rangle = -dt^2 + e^{\sqrt{\frac{\Lambda}{3}}t - \frac{12\pi G\hbar\omega_k}{(L^3\Lambda)}}\delta_{ab}dx^a dx^b$$

Now take Λ → 0 and renormalize

With n=1 we have

$$\omega_k = Z(\frac{L}{l_{Pl}})\omega_R, \qquad Z(\frac{L}{l_{Pl}}) = \frac{\alpha}{12\pi G}(\frac{L}{l_{Pl}})^n$$

$$\langle k|ds^2|k\rangle = -dt^2 + e^{-\alpha l_{Pl}\hbar\omega_R}\delta_{ab}dx^a dx^b$$

Thus, we have derived an energy dependence to the metric,

$$\langle k|ds^2|k\rangle = -dt^2 + e^{-\alpha l_{Pl}\hbar\omega_R}\delta_{ab}dx^a dx^b$$

ie the spatial metric is $h_{ab}(E) = \delta_{ab}(1 - \alpha E/M_P + ...)$

This implies a modified energy momentum relation

$$E^2 = h^{ab}(E)p_a p_b + \mu^2$$

And an energy dependent speed of light

$$v = c(1 + 2\alpha E/M_P + ...)$$

Note, this is DSR and not Lorentz breaking because we have implemented the Hamiltonian constraint that says that there is no preferred slicing of spacetime.

Current active directions to look out for:

Spin foam models:

- •Low energy limit of spin foam models
- •N point functions
- •Renormalization group
- •Matrix models (Group field theory)
- •Deformed symmetry

Cosmology

- •Modified gravity models from extended Plebanski (Krasnov)
- •Odd parity effects in the early universe (Immirzi parameter)
- •models of dark energy
- •inhomogeneities by extending LQC, CMB etc.
- •unimodular gravity (cosmological constant problem and issue of time)

Unification

- •extended Plebanski
- •topological excitations

Conclusions:

General relativity is a gauge theory, closely related to topological field theory (same for Sugrav)

Loop quantum gravity provides a rigorously defined method for quantization of diffeomorphism invariant gauge field theories including these.

For GR in 3+1:

Existence and uniqueness theorems for quantum kinematics, hilbert space, etc.

Discretness of area, volume etc. rigorously shown.

Rigorous solutions to all quantum constraints

Closed form expressions for path integrals

Some results on propagators N point functions,

Some results on semiclassical limit

Models: well tested on 2+1 gravity, cosmological models etc.

Matter can be added by extending G but could also be emergent.

Current priorities:

The path integral:

- Show that the dominant histories are triangulations of smooth spacetimes
- Show that the classical Einstein equations emerge
- Study the 3,4 and higher point functions.
- Renormalization group
- Extend to unifications with matter

The ground state:

- Construct a quantum hamiltonian and show it is positive
- Construct the ground state beyond the semiclassical approximation
- Study the symmetries of the ground state and its excitations:
  Is Lorentz invariance broken or deformed?
  Do the chiral excitations represent massless fermions?

Phenomenology:

- Incorporate inhomogeneities in quantum cosmological models
  make predictions of corrections to CMB spectra
- Parity breaking effects in the early universe (TB mixing)
- Mechanisms for dark energy

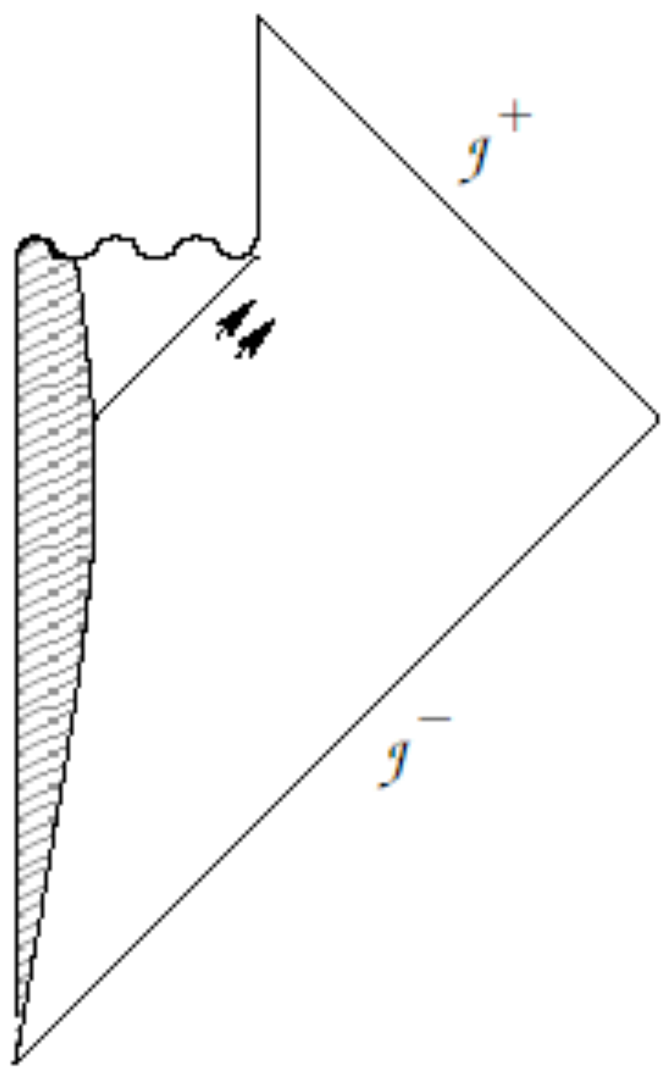
current progress: fast, moderate, slow

Unitarity, information loss and all that

The standard scenario
(Hawking...)

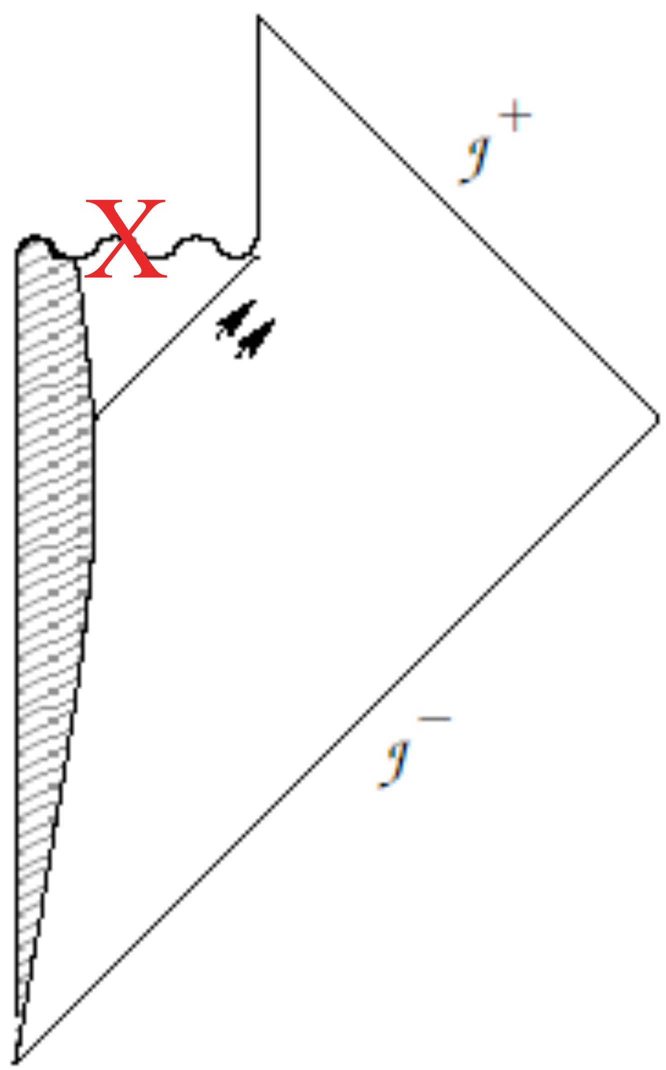
There seems to be a paradox.

Where can the information go?



Unitarity, information loss and all that?

The standard scenario
(Hawking...)



There seems to be a paradox.

Where can the information go?

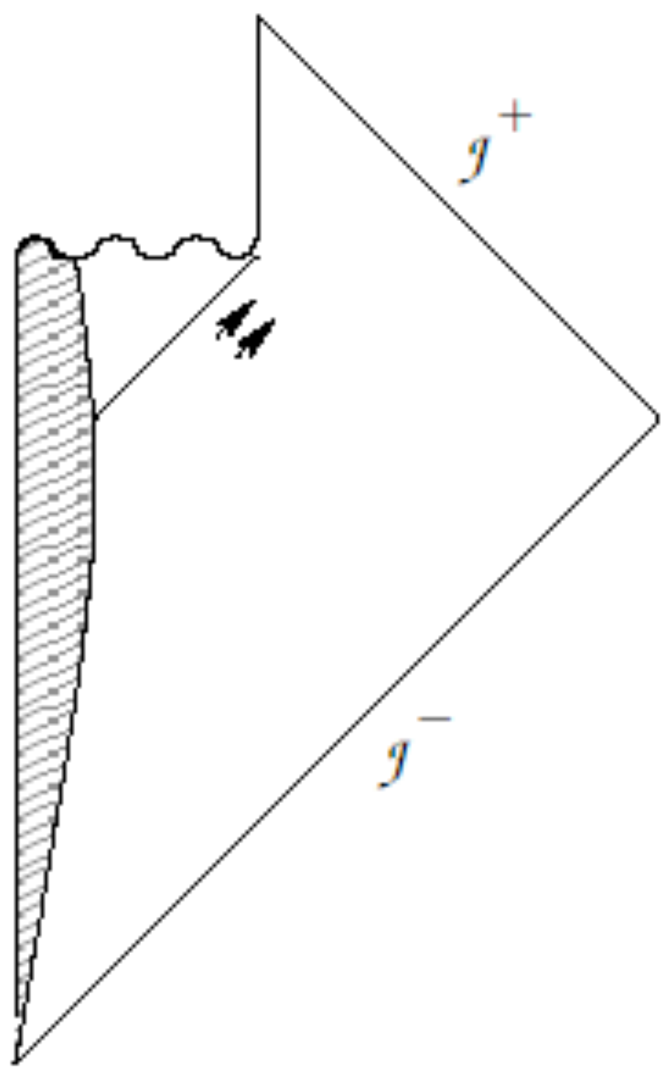
But quantum gravity effects are shown to eliminate the singularity

Modesto gr-qc/0407097, 0504043
Husain & Winkler gr-qc/0410125
Ashtekar & Varadarajan
Ashtekar & Bojowald gr-qc/0504029

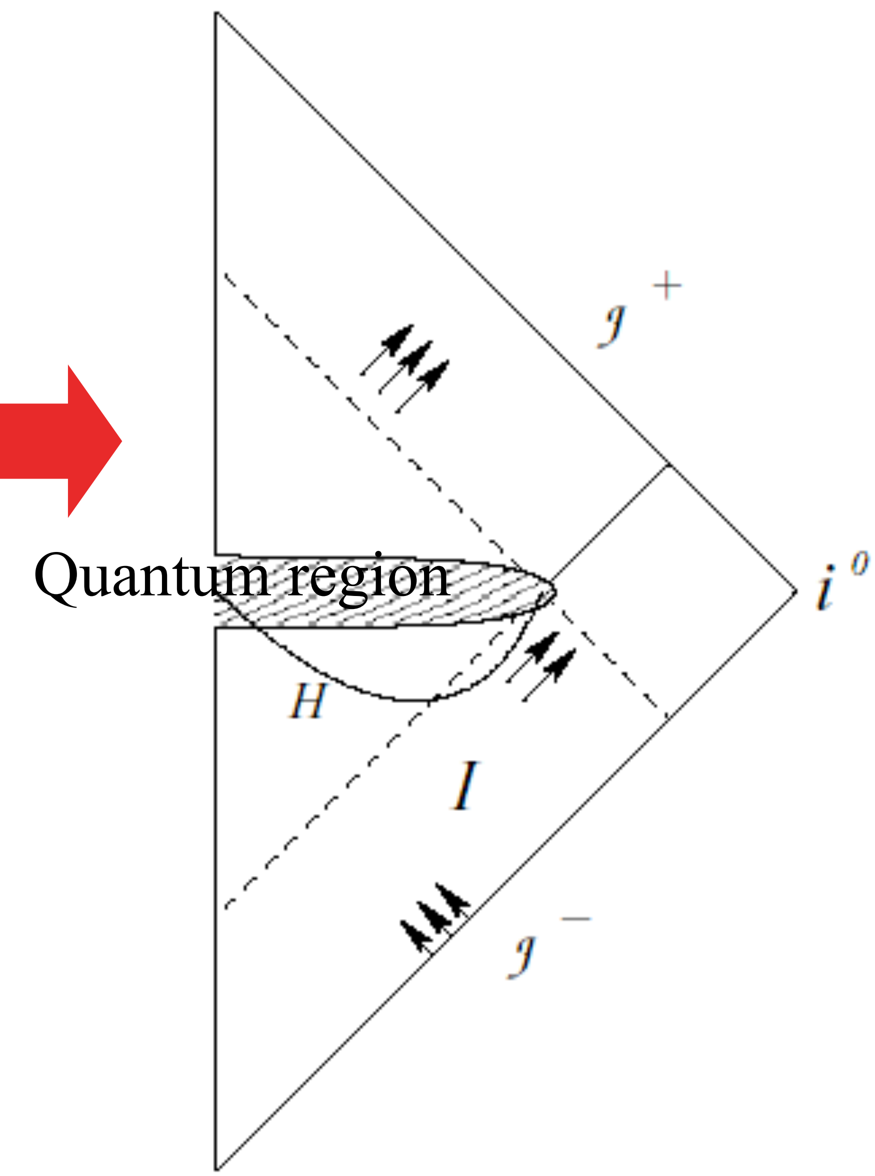
What then??

Unitarity, information loss and all that?

The standard scenario
(Hawking...)



Quantum singularity resolution:
(assuming no permanent baby universe
and finite evaporation time)



Unitarity restored!

What if the black hole doesn't evaporate in finite time?

We get a permanent "baby universe"

But no problem with basic principles.

Information is conserved, so long as all of $\mathcal{I}^+$ is taken into account.